

## Refocusing the Courses Below Calculus: A National Initiative

Sheldon P. Gordon  
Farmingdale State University of New York  
Farmingdale, NY 11735  
gordonsp@farmingdale.edu

According to the 2000 CBMS Study of Undergraduate Programs in the Mathematical Sciences [1], some 2,000,000 students take college algebra and related courses annually. However, according to other data that is emerging [2, 3, 4], these courses do not work for several reasons:

1. Although these courses are designed to prepare students for calculus, only a very small percentage, typically on the order of 10%, of the students who take college algebra ever go on to start Calculus I.
2. These courses do not adequately prepare those students who do go on to subsequent mathematics courses such as statistics, business calculus, and quantitative methods.
3. These courses do not serve the present-day mathematical needs of most other quantitative disciplines, where a deep level of conceptual understanding of the mathematics is deemed more valuable than a very high level of facility in manipulating symbols.
4. The courses do not provide students with the intellectual skills and mathematical understanding needed in the workplace or that will allow them to be effective citizens.
5. At most schools, these courses have unacceptably high DFW rates, typically on the order of 50%, though often much higher. If these courses met the needs of the majority of students taking them, this might be acceptable. But the courses do not meet the needs of the students, so these results should not be acceptable.

A growing part of the mathematics community, particularly the professional organizations involved in mathematics education (MAA, AMATYC, and NCTM), are becoming very concerned about these issues. Between October 2001 and February 2002, there were four special invited conferences – *Rethinking the Preparation for Calculus*, *Reforming College Algebra*, the *Forum on Quantitative Literacy (QL)*, and CRAFTY's *Curriculum Foundations (CF) Summary Workshop*. (CRAFTY had previously run a series of 11 workshops in which leading educators from 17 quantitative disciplines met to inform the mathematics community of the needs of their students today. The summary workshop was held to unify the suggestions from the individual workshops.) The first three conferences focused on the needs of students in courses below calculus. Although the *CF Workshop* did not address these courses specifically, the recommendations from most disciplines focused on college algebra and precalculus because they provide the mathematical foundation for students in those disciplines.

A series of meetings were subsequently held that brought together principals from each conference to identify the common philosophy among the four groups. The goal was to channel the efforts from the four groups into a unified, national initiative to rethink this portion of the curriculum. The first working meeting of this ad hoc group, held at the MAA headquarters in April 2002, was called to:

1. Identify the common elements from among the four groups;
2. Carefully delineate the differences between them;

3. Prepare a formal report on the group's thinking about the courses below calculus to provide guidance for CUPM on this part of the curriculum, as part of CUPM's latest set of recommendations on the mathematics curriculum. It would also inform the writing team that is about to start rewriting the *AMATYC Crossroads Standards*

4. Plan toward a national summit conference similar to the *Calculus for a New Century* symposium that launched the effort to revitalize the teaching of calculus 15 years ago.

A second meeting of the ad hoc group was held at the 2002 MathFest.

Either in attendance at the meetings or participating in the discussions prior to and following the meetings were principals from each of the four conferences. There were also representatives from various MAA committees, including the Task Force on the First College Level Mathematics Course, CUPM, CRAFTY, the Committee on QL, the Committee on Two Year Colleges, and the Committee on Articulation and Placement. The presidents of AMATYC and NCTM, the head of the *AMATYC Standards* writing team and the executive director of MSEB were also involved. The meetings focused solely on the courses below calculus – college algebra and precalculus and the relationship between them and quantitative literacy.

There is total agreement among the 30 members of the ad hoc committee that college algebra and precalculus courses, as presently constituted, do not work for the overwhelming majority of students and do not serve their needs. All participants at the four conferences share the identical sentiments. Most students take these courses only to fulfil general education requirements that are imposed (and often prescribed in extreme detail) by people and groups outside the mathematics department. The ad hoc committee believes that:

1. These courses should have a solid algebraic spine, but that algebraic techniques should not be the focus of the courses.
2. These courses should have a strong emphasis on conceptual understanding and be deep intellectual experiences for the students.
3. It is at least as important to prepare students conceptually for succeeding mathematics courses as it is to prepare them algebraically.
4. These courses should focus on mathematical modeling and realistic problem solving, and that interpretation of results should be a vital component of any applied problem.
5. Data analysis should be an integral part of all of these courses and should be used to connect the mathematics to its use in most other quantitative disciplines.
6. Technology has an important and meaningful role to play in both the teaching and learning of mathematics.
7. The development of writing and communication skills should be an important and significant aspect of these courses.
8. The quantitative literacy theme should permeate all of these courses.

The meetings also addressed some longer term strategies to bring about a climate in which change in the courses below calculus could take place. The ad hoc committee developed a three-year plan culminating in a national summit conference to be convened by the National Academy of Sciences. This conference would launch a movement to rethink and refocus these courses that would be analogous to the effect of the *Calculus for a New Century* conference in launching the effort to revitalize calculus. However, there are some significant differences

between the planned initiative and the efforts to revitalize calculus during the 1990s. First, the changes proposed for calculus did not significantly change the content of the course – they did introduce some new topics; they changed the focus in the course to achieve a better balance between graphical, numerical, and symbolic approaches; and they introduced the use of technology to support both the teaching and learning of mathematics. But a “reform” calculus course was clearly recognizable as a calculus course by anyone in the mathematics community.

Some of the proposed changes to the courses below calculus go substantially further in terms of changing the very nature of the courses. Perhaps the greatest challenge to be faced is changing some very deep-rooted beliefs, both within and without the mathematics community. People who think of college algebra as consisting primarily of a collection of algebraic techniques to be practiced and mastered may not recognize some of the alternative courses as being *algebra* courses. NCTM has worked for years to broaden the meaning of algebra to encompass all types of algebraic reasoning and representations as well as symbolic operations. The same kind of effort will be needed at the college and university level.

Another major difference between calculus renewal and the proposed initiative to refocus the courses below calculus is that the former was basically an academic effort – the key was to convince other mathematicians and some people in allied disciplines of the need to change some aspects of calculus. However, the proposed initiative necessarily extends well beyond the academic arena. In some states, general education requirements, including the specific course content and allowable textbooks, are specified by state education departments and even state legislatures. In many university systems, the courses are specified by academic senates or other external bodies. At many institutions, particularly two year colleges, transfer and articulation agreements limit the changes that can be made in courses. There are many individuals, who are subjected to such external requirements, who express tremendous frustration at not being able to change their courses to better serve the needs of their students. One major challenge is to convince these external bodies to change some of the requirements that they have laid down.

Another difference between calculus renewal and a movement to refocus the courses below calculus is that the former basically started at ground zero. The initial stages of the calculus movement involved the development and testing of new materials that implemented the ideas being discussed. As one unanticipated outgrowth of the effort at revitalizing calculus, various individuals have already developed a variety of alternatives to the traditional courses below calculus. The problem is that there are many barriers that prevent the wide adoption of these materials. The proposed initiative is intended to find ways to circumvent these barriers.

The initiative to refocus these courses must be a collaborative effort among the MAA, AMATYC, and NCTM, since each has a significant interest in these courses. However, a collaboration of this scale and complexity is unprecedented. Thus, it is necessary to develop an appropriate structure that will satisfy the needs of the three organizations as well as to identify the special roles that each organization will play. Once a structure is in place, the working group will have an official mandate from the three societies to approach various groups (governmental and others) outside of the mathematics community that need to be persuaded to encourage or simply allow a new focus for these courses.

The next step is to conduct a reasonably large-scale data collection project to collect and analyze information on the student population that is actually taking these courses.

1. Who are the students who take these courses and why do they take the courses? What are their majors? What requirements do the courses fulfil?
2. Where do the students come from? Do they come from the prerequisite courses given at the same institution, from prerequisite courses at a feeder college, or from high school? How many are actually repeating the course?
3. How do the students do in these courses?
4. What subsequent math courses do they take and how do they do in the follow-up courses?
5. How do the students do in subsequent courses in other quantitative disciplines?

Such data have been collected at several institutions. Steve Dunbar [2] has been conducting such a study at the University of Nebraska for more than 12 years, tracking each of the approximately 120,000 students who have taken mathematics. He has found that, of the students who successfully complete their college algebra course, only about 10% ever go on to start Calculus I; under 1% go on to start Calculus II; and virtually none has started Calculus III. Mercedes McGowen [3] has found very similar results at William Rainey Harper College, a large two year institution. This certainly belies the underlying philosophy on most campuses that college algebra courses are intended to prepare students for mainstream calculus.

As mentioned before, a wide variety of projects have already developed effective and innovative materials that bring a new vision to precalculus, college algebra, quantitative reasoning, and developmental algebra. Each project has its own unique vision, but each vision is clearly in the spirit of what the ad hoc committee believes is appropriate for the students who take these courses. These projects are also consistent with the visions that underlie both the AMATYC *Crossroads* Standards and the NCTM Standards for courses at these levels. Each project has the flexibility to be easily adapted to many different settings and to the needs of many different groups of students and related disciplines. What we need to do is:

1. Identify a variety of institutions that have refocused their college algebra and related courses to reflect the kinds of changes that need to be brought about. These institutions would serve as model programs to prove the existence of successful efforts and to provide specific information on student performance in such courses and in succeeding courses in mathematics and other fields.
2. Stimulate the adaptation and implementation of existing projects at new sites.
3. Stimulate the expansion of model programs at institutions that have already implemented new versions of courses in one or more courses.
4. Stimulate the development and implementation of several new model projects through a series of grants that could be awarded as part of the funding for planning and development phases of this initiative, as discussed below.
5. Track student performance and attitudes throughout these model projects and programs over a several year period to develop some specific data to help in on-going efforts to develop the national initiative.

Perhaps the greatest obstacle to widespread change in the courses below calculus is the attitude that they exist to prepare students for mainstream calculus. What is therefore needed is a

comprehensive effort designed to influence members of the community to try new versions of the courses themselves or to allow and encourage other members of their departments to do so. These efforts should include:

1. Organize panel sessions, contributed paper sessions and poster sessions at national and regional meetings of MAA, AMATYC, NCTM, and ICTCM.
2. Produce volumes in the MAA Notes series and similar series. Some recent and forthcoming volumes that address these issues are (a) The volume of the proceedings of the *Rethinking the Preparation for Calculus* conference edited by Nancy Baxter Hastings, (b) the final results of the *Curriculum Foundations* project being edited by Susan Ganter and Bill Barker, (c) a series of volumes coming out of the *Forum on Quantitative Literacy* being edited by Lynn Steen and Bernard Madison, and (d) the CUPM curriculum recommendations.
3. Provide extensive faculty development activities at a national, a regional, and a local level to assist people in giving new versions of these courses for the first time, with special attention to part time faculty and graduate TAs.
4. Rethink the mathematical training of prospective teachers, who often are required to take college algebra as their final mathematics experience.
5. Influence state agencies and university system-wide bodies that mandate the courses students must take and what the course content must be.
6. Affect transfer and articulation agreements between individual schools and across university-wide systems.
7. Influence the testing industry to develop and implement a new generation of placement and related tests that better reflect the content of NCTM *Standards*-based curricula and the refocused versions of college algebra offerings.
8. Develop links to other disciplines, most likely the social and life sciences, to gain their support, which will be as critical to this initiative as the support of the engineering disciplines was to the calculus effort. Simultaneously, develop links to business/industry/government to seek their support from the point of view of preparing students for today's workplace and for effective citizenship.

Clearly, this is a huge challenge and a huge undertaking. But, each year that passes means that another group of 2,000,000 students are being poorly served by their mathematics courses.

## References

1. Lutzer, David J, James W. Maxwell, and Stephen B. Rodi, 2000 CBMS: Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States, American Mathematical Society, available at <http://www.ams.org/cbms/>.
2. Dunbar, Steve, *Enrollment Flows to and From Precalculus Courses*, in *Rethinking the Road Toward Calculus*, Nancy Baxter Hastings, et al, editors, MAA Notes, 2003 (to appear).
3. McGowan, Mercedes, *Who Are the Students Who Take Precalculus*, in *Rethinking the Road Toward Calculus*, Nancy Baxter Hastings, et al, editors, MAA Notes, 2003 (to appear).
4. Gordon, Florence, *Assessing What Students Learn: Reform versus Traditional Precalculus and Follow-up Calculus*, in *Rethinking the Road Toward Calculus*, Nancy Baxter Hastings, et al, editors, MAA Notes, 2003 (to appear).