Rethinking the Preparation for Calculus

By Jack Narayan, Sheldon Gordon, and Darren Narayan

On October 4, just three weeks after the September 11 attack, 55 mathematics educators participated in an invited conference, funded by the National Science Foundation and the Calculus Consortium for Higher Education (CCHE), to rethink the preparation for calculus. The fact that no one cancelled and five extra participants were accommodated at the last moment is a testimony to the dedication of this group and the importance of precalculus in the mathematics curriculum.

The impetus for the conference included: the changes that have taken place in calculus over the last decade; the significant changes being called for in college algebra as part of a major MAA initiative; the dramatic changes that are taking place in mathematical preparation in high school; and the implications of new technologies that provide a wider selection of mathematical tools for both the teaching and learning of mathematics. Together, these pressures make it an ideal time to rethink precalculus.

Keynote speaker Lynn Steen cited data from the recent CBMS and AMS surveys and noted "precalculus (and its alter ego college algebra) is the single most common mathematics course in undergraduate education." He posed twenty questions suggesting "an overwhelming agenda for a course of enormous importance, but a course that is all but invisible to the mathematical community." He asked "Does the mathematical profession now consider precalculus a challenge worth working on, or do they still see it as a peripheral problem best ignored? Where does precalculus fit into the agenda of mathematics, or science, and of our nation?" He concluded, "Rethinking precalculus may lead to some surprising conclusions."

Mercedes McGowen and Steve Dunbar presented an analysis of the enrollment in mathematics courses at two and four-year colleges and at universities over the past 20 years in an attempt to answer the questions: "Who are the undergraduate students who enroll in precalculus courses? What courses do students take after completing a precalculus course?" The data provide evidence that the present precalculus curriculum is not meeting the needs of *most* of the students enrolled in precalculus courses. The data also indicate that precalculus and remedial (developmental) courses are serving as filters that block many students from attaining their educational goals.

The major themes for the conference included: *Transition from High School, Changes in College Algebra, Precalculus Reform Projects, Technology, Implementation Issues, Research in Student Learning, and Influencing the Mathematics Community*. Invited position papers for each theme were presented and discussed and participants identified challenges and made recommendations.

The discussions were based on a series of basic principles about precalculus courses. Precalculus courses serve two distinct student populations: the overwhelming majority for whom precalculus is a terminal course and the relatively small minority for whom it is a gateway to higher mathematics. We need to identify and meet the needs of both populations.

Precalculus courses need to prepare students for calculus both *conceptually* and *algebraically*. It is not enough just to emphasize developing manipulative skills; students need help to develop the conceptual skills needed to understand and apply the basic calculus concepts. Very few students have the ability to develop those conceptual connections on their own.

Calculus is no longer the first mathematics course that is considered a prerequisite for courses in other quantitative disciplines. Precalculus and college algebra are now prerequisites for (non-calculus-based) courses in almost all fields. The mathematical needs of those fields are often not satisfied by standard, algebra skills-oriented precalculus/college algebra courses.

Students need to see an emphasis on mathematical modeling to learn how mathematics is connected to the real world. The basic mathematical concepts and methods should be developed in context, to help the students transfer their learning outside the mathematics classroom. Precalculus courses should also help students learn to use modern technology wisely and appropriately.

Current research into the learning process has much to tell us about how students acquire fundamental precalculus (mathematical) concepts. Only a small minority of students learn mathematics the way we did.

The primary recommendation from the conference is the need to collect extensive data on who takes precalculus (and college algebra) courses and why. We need to know the success rates in these courses nationwide, which successor courses the students actually take, and how they do in those successor courses. The conference participants felt that such data is critical for convincing the mathematics community at large that these courses need to change, as well as to acquaint potential funding agencies of the magnitude and implications of the problem. Moreover, the participants felt that any efforts to rethink precalculus should involve high school mathematics teachers and faculty in client disciplines.

The Steering Committee is currently preparing an executive summary of the outcomes of the conference. It is also working with the MAA for the publication of the papers and discussions resulting from the conference.

Calculus reformists stressed that calculus should be "a pump, not a filter". Taking precalculus should be a positive experience for *all* students, not just the handful that actually go on into mathematically intensive fields. But this requires rethinking precalculus because a pump is only as good as the motor behind it. ■

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